## Efficient calculation of electron vacuum polarization contributions to energy levels of two-body muonic atoms and ions

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Electron vacuum polarization gives the dominant correction to the energy levels of muonic atoms such as muonic hydrogen and muonic deuterium. In this talk, we present new tools that simplify the calculation of vacuum polarization corrections and lead to improved precision. Specific developments include a convenient form for the three-loop irreducible vacuum polarization correction based on a recent analytic calculation of the three-loop spectral density [1, 2] and new analytic expressions for a number of the integrals that occur in vacuum polarization calculations. Analytic results are given for the one-loop and some of the two- and three-loop corrections that are valid for all states.

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