On the tenth-order QED contribution to the electron anomalous magnetic moment

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The discrepancy of approximately 5σ between the two results for the tenth-order QED contribution to the electron $g-2$ has been recently resolved[1]. We report the details of this resolution.

The contribution from the tenth-order QED Feynman vertex diagrams without fermion loops was first reported in 2012 and later updated by the group led by T. Kinoshita, referred to as AHKN. No alternative calculation existed until S. Volkov independently evaluated the contribution from the same set of Feynman diagrams in 2019 and again in 2024[2]. Both AHKN and Volkov used Feynman parametric representations to express the integrands derived from Feynman diagrams and then numerically evaluated the integrals using the Monte-Carlo integration method. The discrepancy of the results of the coefficient of $(\alpha/\pi)^5$ between AHKN and Volkov was found to be 0.78 ± 0.15, corresponding to a significance of 5.2σ.

In AHKN's calculation, the Feynman vertex diagrams sharing the same photon corrections were summed using the Ward-Takahashi identity. As a result, 389 independent integrals collectively represent the contribution of 6,354 Feynman vertex diagrams. Ultraviolet (UV) divergences were handled by counterterms generated through the $\mathbb{K}$ operation, a power-counting rule in Feynman parametric space.

In contrast, Volkov directly calculated 3,213 vertex diagrams, which are independent under the time-reversal symmetry of QED. The UV counterterms were carefully chosen to ensure that the wave-function and vertex renormalization constants satisfy the Ward-Takahashi identity.

Because of differences in calculation methods, a direct diagram-by-diagram comparison between AHKN and Volkov is not straightforward. To bridge this gap, we expressed the difference in the Ward-Takahashi summed contributions in terms of the symbolic forms involving the lower-order quantities. Once the numerical values of these lower-order quantities were obtained, we performed a numerical comparison of the 389 results between the two approaches.

No significant discrepancies were found for individual results. However, the numerical differences in the 98 Ward-Takahashi summed contributions, originating from diagrams sharing a common structure, were not randomly distributed. The accumulation of these differences led to the observed 5σ discrepancy.

To address this, a recalculation with increased statistics in the Monte Carlo integration was performed for AHKN's 98 integrals. By replacing the previous values with the updated ones, we obtained a revised result, which is consistent with Volkov's result, thereby resolving the discrepancy.

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References

[1] T. Aoyama, M. Hayakawa, A. Hirayama, and M. Nio, *Phys. Rev. D* **111**, L031902 (2025).
[2] S. Volkov, *Phys. Rev. D***100**, 096004 (2019); **110**, 036001 (2024).